DISCOUNTING AND CAPITALIZATION

EXERCISE 1

The expected cash flows of an investment project are:

-1000 600 720

Questions:

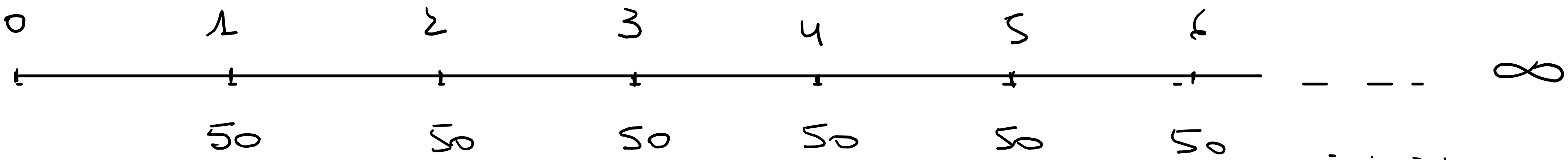
a) Calculate its GPV and NPV for a discounting rate of 10%.

$$\boxed{GPV} = \frac{600}{(1+0'1)} + \frac{720}{(1+0'1)^2} = 1.140'495 \rightarrow \text{MARKET VALUE}$$

$$\boxed{NPV} = -1000 + \frac{600}{(1+0'1)} + \frac{720}{(1+0'1)^2} = 140'495 \rightarrow \text{PROFIT}$$

EXERCISE 2-

Calculate the present value at 10% of a perpetual cash flow of 50 €.



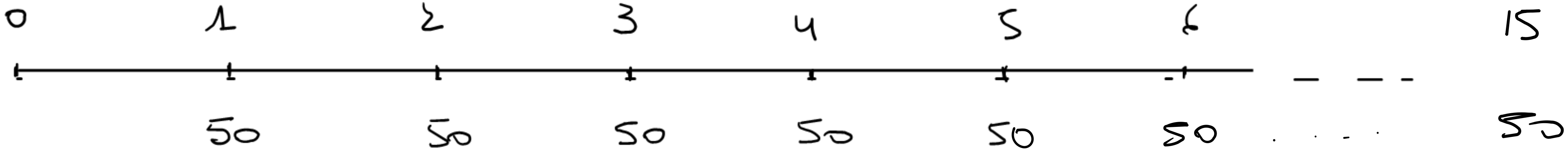
$$PV = \frac{a}{K} = \frac{50}{0'10} = \boxed{500 \text{ €}}$$

EXERCISE 3-

Calculate the present value at 10% of a constant cash flow stream of 50 € during:

- 15 years
- 25 years
- 35 years
- 50 years
- 80 years

$$PV = \frac{a}{k}\left(1 - \frac{1}{(1+k)^n}\right)$$



$$PV(n=15) = \frac{50}{0'10}\left(1 - \frac{1}{(1'10)^{15}}\right) = 380'303975\ €$$

$$PV(n=25) = \frac{50}{0'10}\left(1 - \frac{1}{(1'10)^{25}}\right) = 453'852001\ €$$

$$PV(n=35) = \frac{50}{0'10}\left(1 - \frac{1}{(1'10)^{35}}\right) = 482'207949\ €$$

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$$PV(n=50) = \frac{50}{0'10}\left(1 - \frac{1}{(1'10)^{50}}\right) = 495'740724 \text{ €}$$

$$PV(n=80) = \frac{50}{0'10}\left(1 - \frac{1}{(1'10)^{80}}\right) = 499'755907 \text{ €} \simeq 500 \text{ €}$$

EXERCISE 4-

Calculate the present value at 10% of a perpetual stream of 50€ at its inception growing at 2.5%.

0	1	2	3	4	5	6	...
	50	50(1'025)	$50(1'025)^2$	$50(1'025)^3$	$50(1'025)^4$	......	∞

$$PV = \frac{a}{r-g} = \frac{50}{0'10 - 0'025} = \boxed{666'\overset{\frown}{66} \text{ €}}$$

EXERCISE 5- Taking into account that the present value of a growing perpetual cash flow is:

$$PV = \frac{a}{k-g}$$

Can we say that if g>k then the present value turns out to be negative? No

Only use if g<k

Otherwise → finite horizon → 90 years f.ex.

RISK AND VALUE

EXERCISE 1-

We have project that has the following flows:

0	1	2	3
-1000	50	50	1050

1- Calculate the value of this project if the discount rate is:

a. k = 7,5%
b. k = 5%
c. k = 2,5%

$$K = rf + P$$

$$K = 5\% \pm P$$

$$7'5\% = 5\% + 2'5\%$$

$$5\% = 5\% + 0$$

$$2'5\% = 5\% - 2'5\%$$

$$PV = GPV$$

a) $PV = \frac{50}{(1+0'075)} + \frac{50}{(1+0'075)^2} + \frac{1050}{(1+0'075)^3} = \boxed{934'99}\ €$ → RISK AVERSE

b) $PV = \frac{50}{(1+0'05)} + \frac{50}{(1+0'05)^2} + \frac{1050}{(1+0'05)^3} = 1000\ €$ → RISK NEUTRAL

c) $PV = \frac{50}{(1+0'025)} + \frac{50}{(1+0'025)^2} + \frac{1050}{(1+0'025)^3} = 1.071'40\ €$ → RISK LOVER.

2- If the risk free rate is a 5%, which are the risk attitudes that correspond to each calculation?

3- Calculate the risk premiums in percentage and in value for each case

SELLER.

a) $rp = K - rf = 7'5\% - 5\% = 2'5\%$. | $934'99 - 1000 = -65'01€$

b) $rp = K - rf = 5\% - 5\% = 0$ | $1000 - 1000 = 0$

c) $rp = K - rf = 2'5\% - 5\% = -2'5\%$. | $1071'40 - 1000 = 71'40€$

4- Explain the meaning of the CERTAINTY EQUIVALENT and give its value for each position.

CERTAINTY EQUIVALENT: Amount of money for which one investor would exchange (sell if he owns it, buy if he does not own it) a risky asset.

$PV(k)$ = CERTAINTY EQUIVALENT ✓

EXERCISE 2-

We want to value a German bond that has the following structure

0	1	2	3	4
-2000	500	500	500	2500

1- How much is this bond yielding to someone that paid the nominal price? 25%.

2- If the flows correspond to a risky asset, how would someone that is risk-neutral value it?

3- Calculate the certainty equivalent of a risky asset that has the same structure for an individual that is risk averse and has a risk premium of a 5%. $\rightarrow K = 25\% + 5\% = 30\%$

4- And for someone that is risk lover and has a risk premium of a -2%?

$\hookrightarrow K = 25\% - 2\% = 23\%$

(2) $$PV = \frac{500}{(1+0'25)} + \frac{500}{(1+0'25)^2} + \frac{500}{(1+0'25)^3} + \frac{2.500}{(1+0'25)^4} = 2000$$ Risk Neutra

(3) $$PV = \frac{500}{(1+0'30)} + \frac{500}{(1+0'30)^2} + \frac{500}{(1+0'30)^3} + \frac{2500}{(1+0'30)^4} = 1.783'38 \text{ €}$$ Risk ave

(4) $$PV = \frac{500}{(1+0'23)} + \frac{500}{(1+0'23)^2} + \frac{500}{(1+0'23)^3} + \frac{2500}{(1+0'23)^4} = 2.097'93 \text{ €}$$ Risk lo